Flow of Dielectric Liquid Over a Gravity Aligned Stretching Sheet

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Abstract

A gravity aligned stretching sheet was considered to analyze the flow of dielectric liquid and the associated heat transfer. Mathematical modeling of the fluid flow consists of equations of continuity, momentum and energy. A Runge-Kutta based shooting strategy was employed to illuminate the system incorporating non-linear ordinary differential equations which represents the system. Velocity and temperature profiles were obtained for a range of parameters. Stretchable materials can benefit from the developed calculation methods.

Keywords: *Dielectric liquid, Electric Dipole, Stretching sheet, RK45 method, Heat transfer, Grashof number.*

1.0 Introduction

Several authors studied boundary layer flows, stretching sheets and viscoelastic liquid. Importance of stretching in extrusion of plastic is studied [1-2]. B C Sakiadis [3] investigated the behavior of boundary layer on continuous solid surface and flat surface. Behavior of laminar and turbulent boundary layers on a moving continuous cylindrical surface was determined by B C Sakiadis [4]. C H Chen [5] presented study of vertical, laminar mixed convection which expands continuously. The velocity and temperature of the sheet were expected to follow a Power Law curve. Using analytical and experimental approaches.

F K Tsou [6] investigated the flow and temperature fields in the boundary layer of a continuous moving surface. L J Crane [7] presented a detailed form for the steady boundary layer flow of an incompressible viscous liquid caused purely by the linear stretching of an elastic sheet moving in its own plane at a velocity which increased linearly with distance from a fixed point. B Siddappa et al. [8] presented solution of equation of motion for boundary layer flow past a stretching plate using

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the crane's problem and Walters' liquid B model's viscoelastic fluid. In the presence of a uniform free stream, R K Bhatnagar et al. [9] studied the creation of a stretched sheet which causes the flow. As a result of pairing a vertical electric field with a vertical temperature gradient, M I A Othman [10] examined the onset of stability within a horizontal layer of viscoelastic dielectric liquid.

P G Siddeshwar et al. [11] discussed the thermal instability that results in a layer of dielectric liquid when its boundary is subjected to synchronous/asynchronous temperature variations. A Abraham [12] investigated convective instability in magnetic fluids and polarized dielectric liquids. P G Siddheshwar et al. [13] studied the boundaries of a layer of dielectric fluid subjected to tiny amplitude time-periodic body force. The layer becomes thermally unstable. The authors obtained a perturbation solution in powers of the amplitude of applied temperature field.

Using a weak nonlinear analysis, P G Siddheshwar et al. [14] examined the influence of time-periodic oscillations of the Rayleigh-Benard system on heat transfer in dielectric liquids. A Abraham et al. [15] used the Galerkin technique to examine Rayleigh-Benard-Marangoni instability in a micro-polar dielectric liquid. As a result of gravity-aligned stretching of an elastic sheet, L S Titus et al. [16] analyzed the flow generated by a magnetic dipole. As part of the study on momentum, heat, and mass transfer behavior, N Sandeep et al. [17] used nanofluid flow embedded with conducting dust particles to study radiation, non-uniform heat source / sink, nanoparticle volume fraction and chemical reactions.

A Majeed et al. [18] analyzed a ferrofluid flow under a magnetic dipole with a suction effect and Soret effect at a stretching sheet under the influence of chemical processes. G Bognar [19] reported finding new results concerning magneto-thermomechanical interactions of viscous ferrofluids with cold walls in the presence of a spatially changing magnetic field. L J Grubka et al. [20] studied heat transfer characteristics of a continuous, stretching surface with variable temperature.

Ferrofluid in a magnetic field has been the focus of several studies in literature examining its heat transfer and flow. The objective of the present research is to examine the gravity aligned stretching effects on the fluid flow and heat transfer in dielectric liquid in the presence of an electric field.

2.0 Mathematical modeling of the dielectric liquid flow

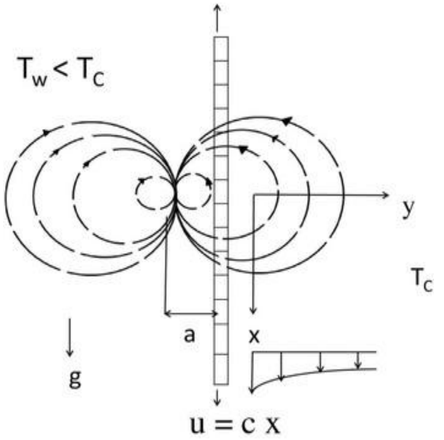

Fig. 1. Schematics of flow of a dielectric liquid

Consider a dielectric liquid that is viscous, incompressible, and electrically non-conducting, being made to flow vertically by an impermeable sheet in two dimensions. Two equal and opposite forces to the sheet are applied along the gravity axis (x-axis) and the flow axis (y-axis). Dipoles are separated by distance from the sheets. In a dipole, the center is located on the y-axis, while the electric field is in the positive x-direction at a distance ‘a’, so that a strong electric field can saturate the dielectric.

A sheet is stretched by maintaining constant temperature T_w below the Curie temperature T_c, while the fluid elements distant from the sheet are kept at a constant temperature $T = T_c$ as shown in Fig. 1 and the behaviour of the system is represented by equations (1-10)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{P}{\rho}\frac{\partial E}{\partial x} + g\beta^*(T_C - T) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + T\frac{\partial P}{\partial T}(u\ \frac{\partial E}{\partial x} + v\ \frac{\partial E}{\partial y}) = \frac{K}{\rho C_P}\frac{\partial^2 T}{\partial y^2} + \frac{K}{\rho C_P}\left[\mu\ \left(\frac{\partial u}{\partial x}\right)^2 + 2\mu\left(\frac{\partial v}{\partial y}\right)^2\right] \tag{3}$$

The notations used in the equations are:

u, v	velocity components along x and y direction respectively
T	fluid temperature
P	polarization

ρ	fluid density
μ	dynamic viscosity
$\nu = \frac{\mu}{\rho}$	kinematic viscosity
C_p	specific heat at constant pressure
k	thermal conductivity
g	acceleration due to gravity
β^*	coefficient of thermal expansion

The boundary conditions for temperature are presented in equation (4)

$$\left.\begin{aligned} &u(x,0) = cx, v(x,0) = 0 \\ &T(x,0) = T_w = T_c - A\left(\frac{x}{L}\right) \text{ in PST} \\ &-k\frac{\partial T}{\partial y}(x,0) = q_w = D\left(\frac{x}{L}\right) \text{ in PHF} \\ &u(x,\infty) \to 0,\; T(x,\infty) \to T_c \end{aligned}\right\} \quad (4)$$

A, D	positive constant
$L = \sqrt{\frac{\nu}{c}}$	charecteristic length
T	temperatre of the fluid
T_c	curie temperatre
T_w	temperature of stretching string

The electric dipole influences the flow of the dielectric liquid because of the electric field. This scalar electric potential is given by equation (5).

$$\emptyset = \left(\frac{x}{(x^2+(y+a)^2)}\right)\frac{\alpha'}{2\pi} \quad (5)$$

The elements of the electric field E are shown in equations (6-9).

$$E_x = -\frac{\partial\emptyset}{\partial x} = \frac{\alpha'}{2\pi}\left(\frac{x^2-(y+a)^2}{x^2+(y+a)^2}\right), \quad (6)$$

$$E_y = -\frac{\partial\emptyset}{\partial y} = \frac{\alpha'}{2\pi}\left(\frac{2x(y+a)}{x^2+(y+a)^2}\right), \quad (7)$$

$$E = \left[\left(\frac{\partial \emptyset}{\partial x}\right)^2 + \left(\frac{\partial \emptyset}{\partial y}\right)^2\right]^{\frac{1}{2}}, \tag{8}$$

$$\left.\begin{aligned} \frac{\partial E}{\partial x} &= -\frac{\alpha'}{2\pi}\left(\frac{2x}{(y+a)^4}\right) \\ \frac{\partial E}{\partial y} &= -\frac{\alpha'}{2\pi}\left(\frac{-2}{(y+a)^3} + \frac{4x^2}{(y+a)^5}\right) \end{aligned}\right\} \tag{9}$$

where, α' is the strength of the electric field at the source.

Linear equation (10) represents variation of polarization P with respect to dielectric constant.

$$P = \varepsilon_0(\varepsilon_r - 1) \tag{10}$$

where, ε_0 *and* ε_r denote the absolute and relative dielectric permittivity respectively.

3.0 Solution Procedure

Andersson [6] assumed non- dimensional variables as shown in equation (11).

$$(\xi, \eta) = \left(\frac{c}{\nu}\right)^{\frac{1}{2}}(x, y), \; (U, V) = \frac{(u,v)}{\sqrt{c\nu}} \tag{11}$$

In prescribed surface temperature (PST) and prescribed surface heat flux (PHF)

$$\theta(\xi, \eta) = \frac{T_c - T}{T_c - T_w} = \begin{cases} \theta_1(\eta) + \xi^2\theta_2(\eta) & \text{in PST case} \\ \phi_1(\eta) + \xi^2\phi_2(\eta) & \text{in PHF case} \end{cases} \tag{12}$$

where, $T_c - T_w = A\left(\frac{x}{L}\right)$ *in PST case,*

$$T_c - T_w = \frac{DL}{K}\left(\frac{x}{L}\right) \quad \text{in PHF case,}$$

Using (9 -12), the boundary layer equations (2 and 3) assume the form given by equations (13 -16).

$$\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0 \tag{13}$$

$$U\frac{\partial U}{\partial \xi} + V\frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2} - \frac{2\beta\xi}{(\eta+\alpha)^4}(\theta_1 + \xi^2\theta_2) + G_r\xi(\theta_1 + \xi^2\theta_2) \tag{14}$$

$$\Pr\left[2U\xi\theta_2 + V(\theta_1' + \xi^2\theta_2')\right] = \theta_1'' + \xi^2\theta_2'' - \lambda\left(\left(\frac{\partial U}{\partial \eta}\right)^2 + 2\left(\frac{\partial V}{\partial \eta}\right)^2\right) \tag{15}$$

Boundary condition (4) now takes the form.

$$\left.\begin{array}{lll} U(\xi,0)=\xi, & V(\xi,0)=0 & \\ \theta_1(\xi,0)=1, & \theta_2(\xi,0)=0 & in\ \ PST \\ & & \\ \phi_1'(\xi,0)=-1, & \phi_2'(\xi,0)=0 & in\ \ PHF \\ U(\xi,\infty)\to 0, & \theta(\xi,\infty)\to 0 & \end{array}\right\} \quad (16)$$

By introducing the stream function $(\xi,\boldsymbol{\eta})=\xi f(\boldsymbol{\eta})$, equation (17) is obtained.

$$U=\frac{\partial\Psi}{\partial\eta}=\xi f^{'}(\eta),\quad V=\frac{\partial\Psi}{\partial\eta}=-f(\eta) \quad (17)$$

In the dimensionless version, equation (17) satisfies the continuity equation (14).

The differentiation with respect to η is indicated by prime. The boundary value is derived when (9), (11), and (17) are used in (14) and (15).

Case (i) Prescribed surface temperature (PST) is given by equation (18-20).

$$f^{'''}-f^{'2}+ff^{''}-\frac{2\beta(\theta_1(\eta))}{(\eta+\alpha)^4}+G_r\theta_1=0 \quad (18)$$

$$\theta_1^{''}-2\lambda f^{'2}+Prf\theta_1^{'}+\frac{2\beta\lambda f(\theta_1(\eta)-\varepsilon)}{(\eta+\alpha)^3}=0 \quad (19)$$

$$\theta_2^{''}-\Pr(2f^{'}\theta_2-f\theta_2^{'})-\lambda f^{''2}-\beta\lambda(\theta_1(\eta)-\varepsilon)\left[\frac{2f^{'}}{(\eta+\alpha)^4}+\frac{4f}{(\eta+\alpha)^5}\right]=0 \quad (20)$$

Several limit conditions are given by equation (21-22)

$$f=0,\quad f^{'}=1,\ \theta_1=1,\ \theta_2=0\ at\ \ \eta=0 \quad (21)$$

$$f^{'}\to 0,\ \ \theta_1\to 0,\ \ \theta_2\to 0\ \ \ as\ \ \ \eta\to\infty \quad (22)$$

Case (ii) Prescribed surface heat flux (PHF) is given by equation (23-25).

$$f^{'''}-f^{'2}+ff^{''}-\frac{2\beta(\phi_1(\eta))}{(\eta+\alpha)^4}+G_r\phi_1=0 \quad (23)$$

$$\phi_1^{''}-2\lambda f^{'2}+Prf\phi_1^{'}+\frac{2\beta\lambda f(\phi_1(\eta)-\varepsilon)}{(\eta+\alpha)^3}=0 \quad (24)$$

$$\phi_2^{''}-\Pr(2f^{'}\phi_2-f\phi_2^{'})-\lambda f^{''2}-\beta\lambda(\phi_1(\eta)-\varepsilon)\left[\frac{2f^{'}}{(\eta+\alpha)^4}+\frac{4f}{(\eta+\alpha)^5}\right]=0 \quad (25)$$

Several limit conditions are given by equation (26)

$$f=0,\quad f^{'}=1,\ \ \phi_1^{'}=-1,\ \ \phi_2^{'}=0\ at\ \ \eta=0 \quad (26)$$

$$f^{'}\to 0,\ \ \phi_1\to 0,\ \ \phi_2\to 0\ \ \ as\ \ \ \eta\to\infty$$

where,

$Pr = \frac{\mu c_p}{k}$ is the Prandtl number,

$\lambda = \frac{c\mu}{\rho k(T_c - T_w)}$ is viscous dissipation,

$\beta = \frac{\alpha' \rho}{2\pi\mu^2}\epsilon_0 e(T_c - T)$ Dielectric interaction parameter,

$\alpha = \left(\frac{c\rho a^2}{\mu}\right)^{\frac{1}{2}}$ is Dimensionless distance,

$Gr = \frac{g\beta^* A}{c^2 L}$ is Grashof number.

At the sheet the dimensionless form of the shear stress τ which is nothing but local skin friction coefficient denoted by C_f is given by

$$C_f = \frac{-2\tau_{xy}}{\rho(cx)^2} = -2f''(0)Re_x^{\frac{-1}{2}}$$

Fixing the surface temperature it is possible to calculate local heat flux and is given by

$$Nu_x = -Re_x^{\frac{1}{2}}[\theta_1'(0) + \xi^2\theta_2'(0)]$$

The shooting technique and the Runge Kutta Fehlberg (RKF45) method are used to solve two point boundary value problems generated by equations (17 - 21) and (18-24). To satisfy the outer boundary requirement, Newton-Raphson's approach is used to alter the trial values of $f''(0), \theta_1'(0), \theta_2'(0), \phi_1'(0)$and $\phi_2'(0)$ iteratively.

4.0 Results and Discussions

The effect of changing the parameters β, Pr and Gr on dielectric liquid flow and heat transfer over a gravity aligned stretching sheet are investigated. These impacts are investigated under the conditions of Prescribed Surface Temperature and Prescribed Surface Heat Flux. $\alpha = 1, \varepsilon = 2 \; and \; \lambda = 0.01$ are used to draw graphs.

If β is increased, an electric field formed by an electric dipole increases the friction on a fluid. This friction acts as a retardant, as indicating lowering velocity in Fig. 2a and flattening $f'(\eta)$. As shown in Fig. 2b, the dielectric interaction parameter β affects the temperature profile. With an increased value of β, the thermal boundary layer is thicker both in the PHF and PST cases.

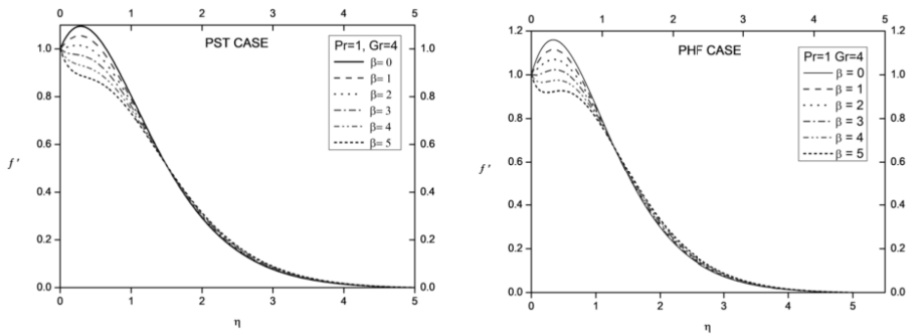

Fig. 2 a) Flow characteristics for various dielectric interaction parameter β

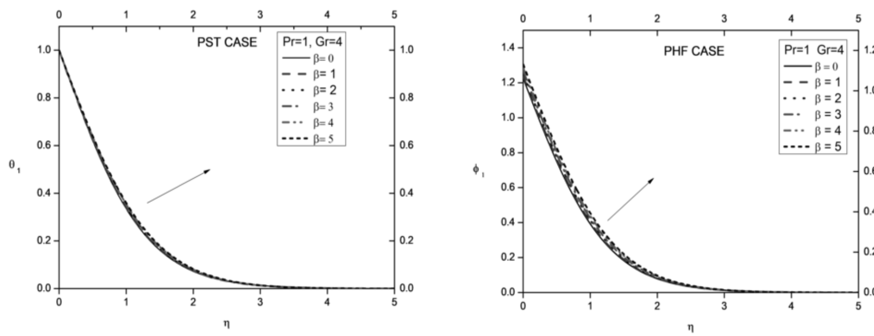

Fig. 2 b). Temperature profile for $\theta(\eta)$ for various values of β in PST and PHF

It is evident in the case of both PST and PHF that rising Pr values reduces horizontal velocity profiles. This is shown in Fig. 3a, which illustrates the influence of Pr values on thermal boundary layers.

According to Fig. 3b, as the Pr number increases, the fluid becomes extremely viscous, and the velocity drops. It can clearly be seen in Fig. 3b that a fluid with a lower Prandtl number reduces heat transmission more efficiently.

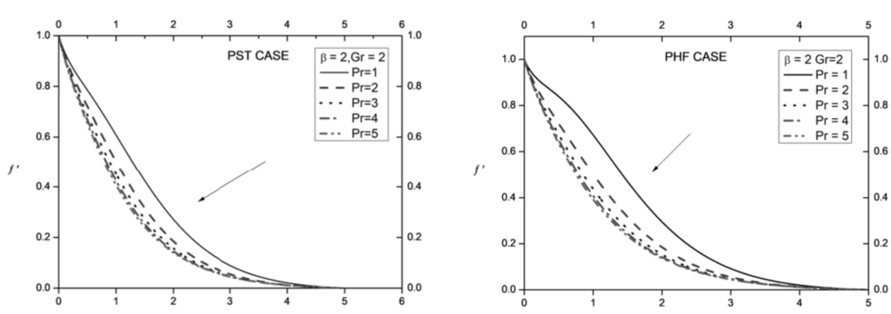

Fig. 3 a). Flow characteristics for various Prandtl number Pr**.**

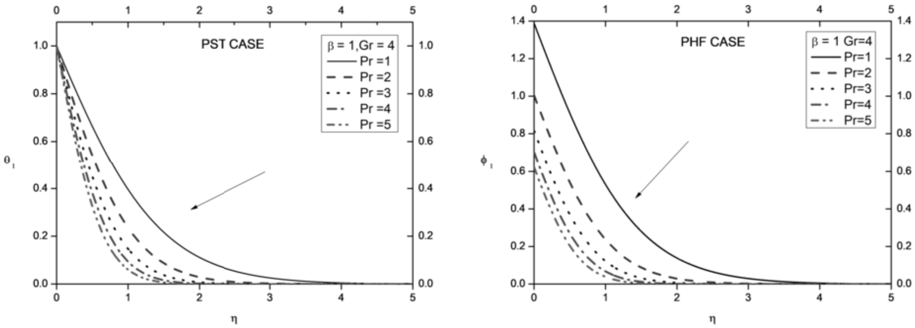

Fig. 3 b). Heat characteristics for various Prandtl number Pr**.**

For PST and PHF in Fig. 4a is the effect of the Grashof number Gr on velocities can be observed. The importance of convection in influencing axial velocity is highlighted by the Grashof number. These curves showed that when Gr grows, the thickness of the momentum boundary layer thickens, by allowing the fluid to flow freely. Vertical stretching sheet cooling produces a favorable pressure gradient that causes fluid in the boundary layer region to accelerate. This illustrates the importance of inertia when compared with viscous forces. The effects of advection are greater than those of conduction when Gr is high and inertial forces always win over viscous forces. Additionally, the flow may have boundary layer characteristics. Figure 4b illustrates influence of Grashof number on heat transmission. This shows Grashof number and Prantl number has same effect on heat transfer.

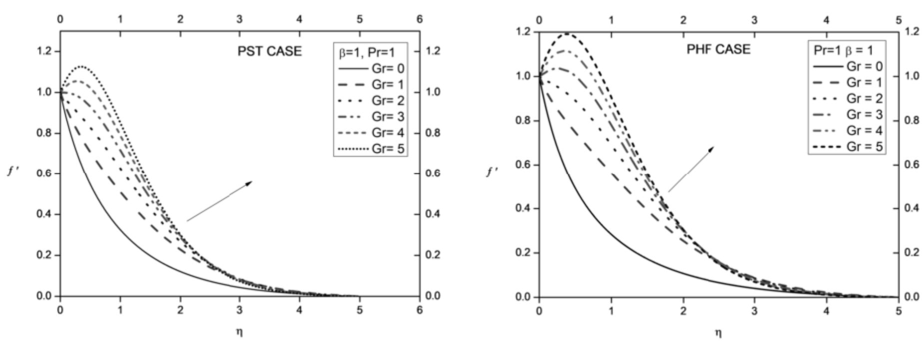

Fig. 4. a) Flow characteristics for various Grashof number Gr.

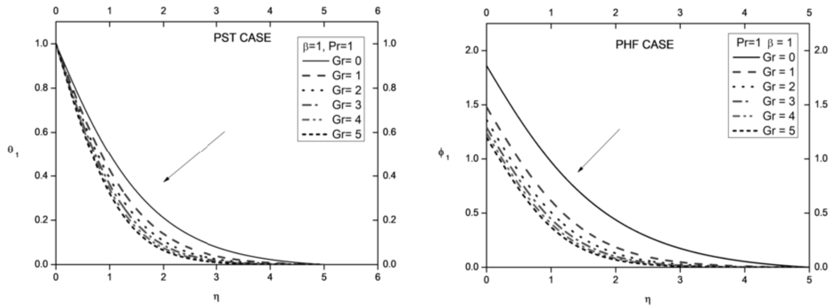

Fig. 4 b) Heat characteristics for various Grashof number Gr.

Table 1 Validation of current findings using previously published works in a restricted number of cases when $\beta = G_r = \lambda = 0$

Table 1. Validation of Results

Pr	L J Grubka et.al [20]	C H Chen et.al [5]	Current findings
1	0.5820	0.58199	0.5819
3	1.1652	1.16523	1.1652
10	7.7657	7.76536	7.7653

5.0 Conclusion

This paper describes the problem of a gravity aligned stretched sheet. The results are concluded as:

- An inclined stretching sheet showed that a dielectric liquid can be successfully used for generating the necessary temperature, which enhances the stretchability.
- Regulation of heat transfer can be achieved by fluids of less Prandtl number.
- Minimum values of Grashof and Prandtl numbers resulted ineffective cooling and improves the flow.

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